MATH 543 - HW 2

Parham Khodadi

February 10, 2026

TB-4.1

Problem

Determine the singular value decompositions (SVDs) of the following matrices (by hand calculation):

$$\text{(a)} \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}, \qquad \text{(b)} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \qquad \text{(c)} \begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad \text{(d)} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \qquad \text{(e)} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

MATLAB code

Listing 1: My MATLAB code for solving this question.

```
%% tb-4.1
fprintf("TB-4.1\n\n");

% Define every matrix
A_a = [3,0;0,-2];
A_b = [2,0;0,3];
A_c = [0,2;0,0;0,0];
A_d = [1,1;0,0];
A_e = [1,1;1,1];

% SVD then display U, S, V
[U_a,S_a,V_a] = svd(A_a);
fprintf("SVD of Matrix (a):\n")
fprintf("U:\n"); disp(U_a);
fprintf("S:\n"); disp(S_a);
fprintf("V:\n"); disp(V_a);
fprintf("\n");

[U_b,S_b,V_b] = svd(A_b);
fprintf("SVD of Matrix (b):\n")
fprintf("U:\n"); disp(U_b);
fprintf("S:\n"); disp(S_b);
fprintf("V:\n"); disp(V_b);
fprintf("\n");

[U_c,S_c,V_c] = svd(A_c);
fprintf("SVD of Matrix (c):\n")
fprintf("U:\n"); disp(U_c);
fprintf("S:\n"); disp(S_c);
fprintf("V:\n"); disp(V_c);
```

```
fprintf("\n");

[U_d,S_d,V_d] = svd(A_d);
fprintf("SVD of Matrix (d):\n")
fprintf("U:\n"); disp(U_d);
fprintf("S:\n"); disp(S_d);
fprintf("V:\n"); disp(V_d);
fprintf("\n");

[U_e,S_e,V_e] = svd(A_e);
fprintf("SVD of Matrix (e):\n")
fprintf("U:\n"); disp(U_e);
fprintf("S:\n"); disp(S_e);
fprintf("V:\n"); disp(V_e);
fprintf("\n");

fprintf("—————————————————\n");
```

Output

```
TB-4.1

SVD of Matrix (a):
U:
     1     0
     0     1

S:
     3     0
     0     2

V:
     1     0
     0    -1


SVD of Matrix (b):
U:
     0     1
     1     0

S:
     3     0
     0     2

V:
     0     1
     1     0


SVD of Matrix (c):
U:
     1     0     0
     0     1     0
```

```
     0     0     1

S:
     2     0
     0     0
     0     0

V:
     0    -1
     1     0


SVD of Matrix (d):
U:
    1.0000         0
         0    1.0000

S:
    1.4142         0
         0         0

V:
    0.7071   -0.7071
    0.7071    0.7071


SVD of Matrix (e):
U:
   -0.7071   -0.7071
   -0.7071    0.7071

S:
    2.0000         0
         0    0.0000

V:
   -0.7071    0.7071
   -0.7071   -0.7071


-----------------
```

TB-4.3

Problem

Write a MATLAB program (see Lecture 9) which, given a real 2×2 matrix A, plots the right singular vectors v_1 and v_2 in the unit circle and also the left singular vectors u_1 and u_2 in the appropriate ellipse, as in Figure 4.1. Apply your program to the matrix (3.7) and also to the 2×2 matrices of Exercise 4.1.

MATLAB code

Listing 2: My MATLAB code for solving this question.

```
%% tb-4.3
fprintf("TB-4.3\n\n");

% Ask for Matrix
A = zeros(2,2);
fprintf("Please input the value for each element in the 2x2 matrix A, below\n");
A(1,1) = input("A_11 = ");
A(1,2) = input("A_12 = ");
A(2,1) = input("A_21 = ");
A(2,2) = input("A_22 = ");

% SVD
[U,S,V] = svd(A);
sigma1 = S(1,1);
sigma2 = S(2,2);
v1 = V(:,1);
v2 = V(:,2);
u1 = U(:,1);
u2 = U(:,2);

% Display SVD
fprintf("Singular values of matrix A:\n");
fprintf("Sigma1 = %.4f\nSigma2 = %.4f\n", sigma1, sigma2);
fprintf("v1 = [%.6g; %.6g], v2 = [%.6g; %.6g]\n", v1(1), v1(2), v2(1), v2(2));
fprintf("u1 = [%.6g; %.6g], u2 = [%.6g; %.6g]\n\n", u1(1), u1(2), u2(1), u2(2));

% Unit Circle + V
t = linspace(0, 2*pi, 31415);
X = [cos(t); sin(t)];
Y = A*X;

figure;
plot(X(1,:), X(2,:), 'k', 'LineWidth', 1.5); hold on; grid on;
quiver(0,0, v1(1), v1(2), 0, 'LineWidth', 2, 'MaxHeadSize', 0.25);
quiver(0,0, v2(1), v2(2), 0, 'LineWidth', 2, 'MaxHeadSize', 0.25);
axis equal;
xlabel('x'); ylabel('y');
title('Unit circle with right singular vectors');
legend('unit circle','$v_1$','$v_2$','Location','best','Interpreter','latex');
exportgraphics(gcf, 'unit_circle_V.eps', 'ContentType', 'vector');


% Ellipse + Sigma*U
```

```
p1 = sigma1*u1;
p2 = sigma2*u2;

figure;
plot(Y(1,:), Y(2,:), 'k', 'LineWidth', 1.5); hold on; grid on;
quiver(0,0, p1(1), p1(2), 0, 'LineWidth', 2, 'MaxHeadSize', 0.25);
quiver(0,0, p2(1), p2(2), 0, 'LineWidth', 2, 'MaxHeadSize', 0.25);
axis equal;
xlabel('x'); ylabel('y');
title('Ellipse with left singular vectors');
legend('ellipse','$\sigma_1 u_1$','$\sigma_2 u_2$','Location','best',
'Interpreter','latex');
exportgraphics(gcf, 'ellipse_U.eps', 'ContentType', 'vector');
```

Output

0.0.1 When using matrix (3.7)

Matrix (3.7) in the book is $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$.

Here's the console output from this code:

```
TB-4.3

Please input the value for each element in the 2x2 matrix A, below
A_11 = 1
A_12 = 2
A_21 = 0
A_22 = 2
Singular values of matrix A:
Sigma1 = 2.9208
Sigma2 = 0.6847
v1 = [0.256668; 0.9665], v2 = [-0.9665; 0.256668]
u1 = [0.749678; 0.661803], u2 = [-0.661803; 0.749678]
```

The following figures (Figures 1 and 2) are generated.

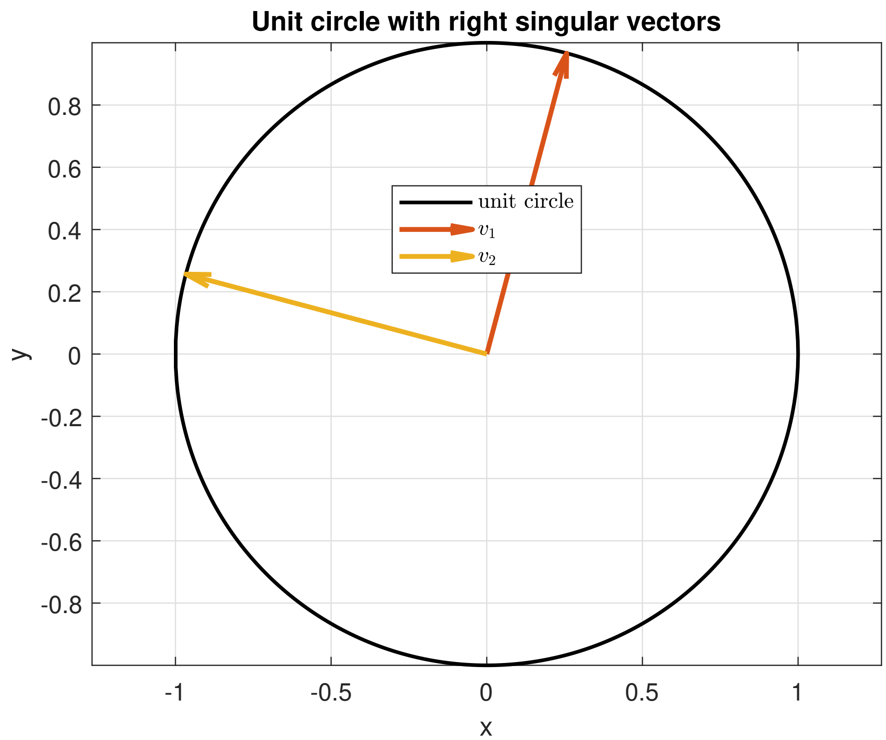


Figure 1: Unit circle with right singular vectors v_1 and v_2.

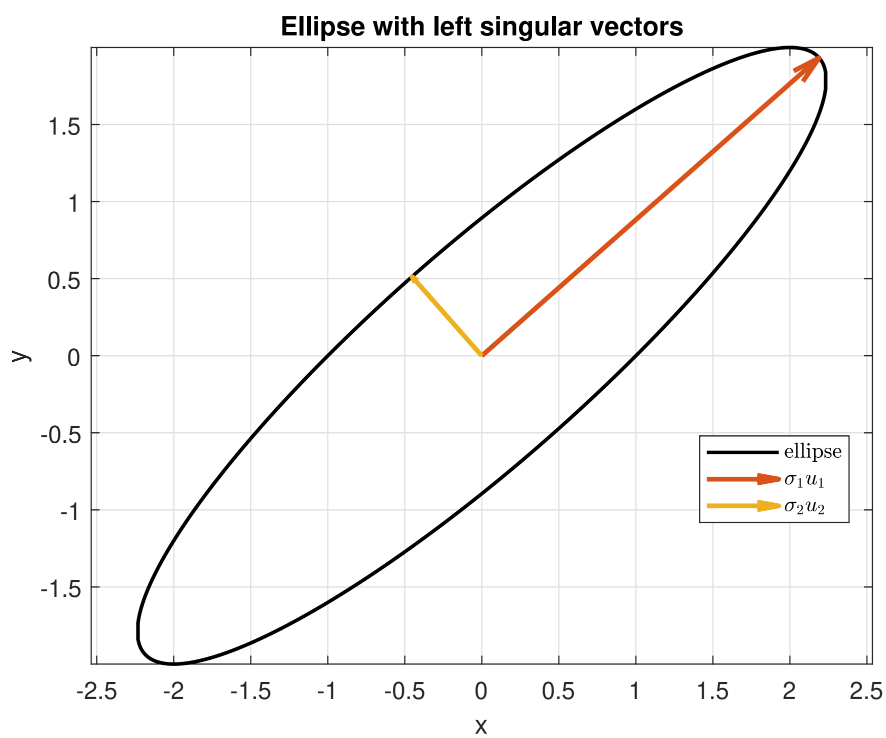
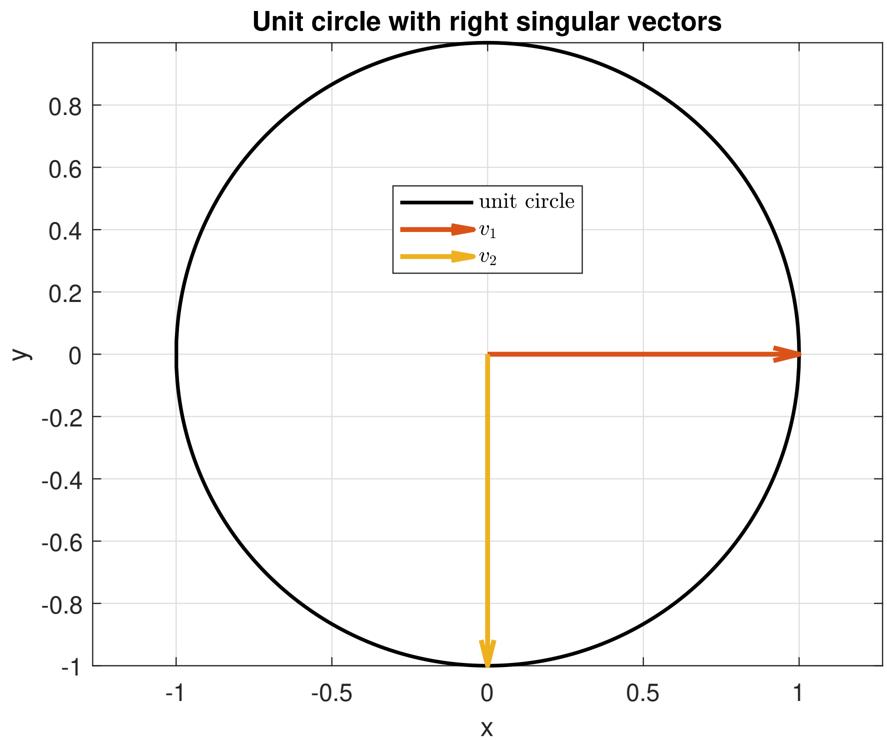
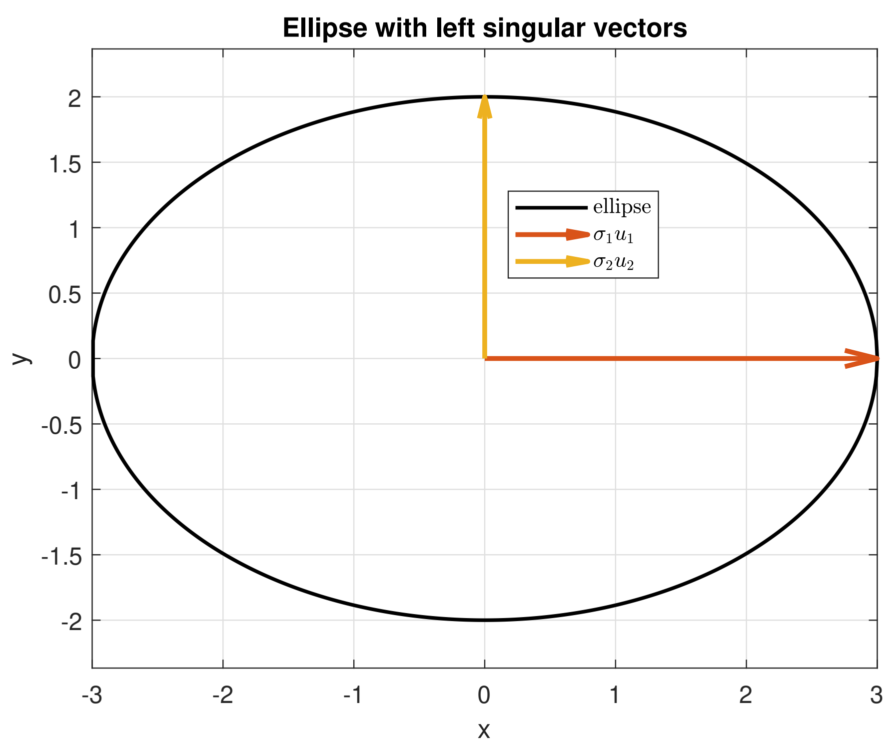


Figure 2: Ellipse with singular values times left singular vectors $\sigma_1 u_1$ and $\sigma_2 u_2$.

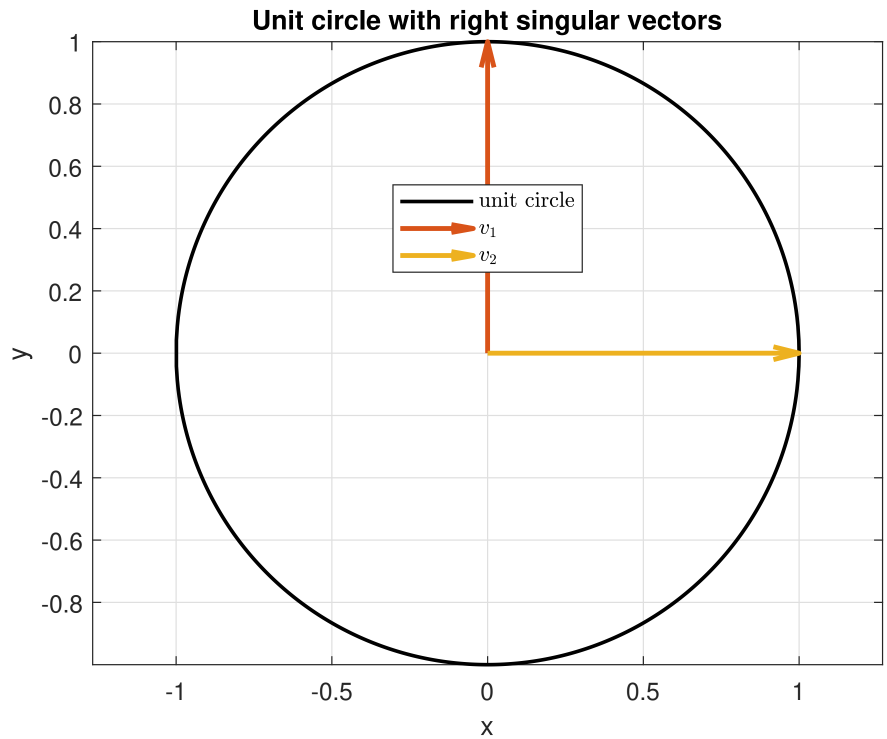
0.0.2 The 2×2 Matrices in Problem 4.1

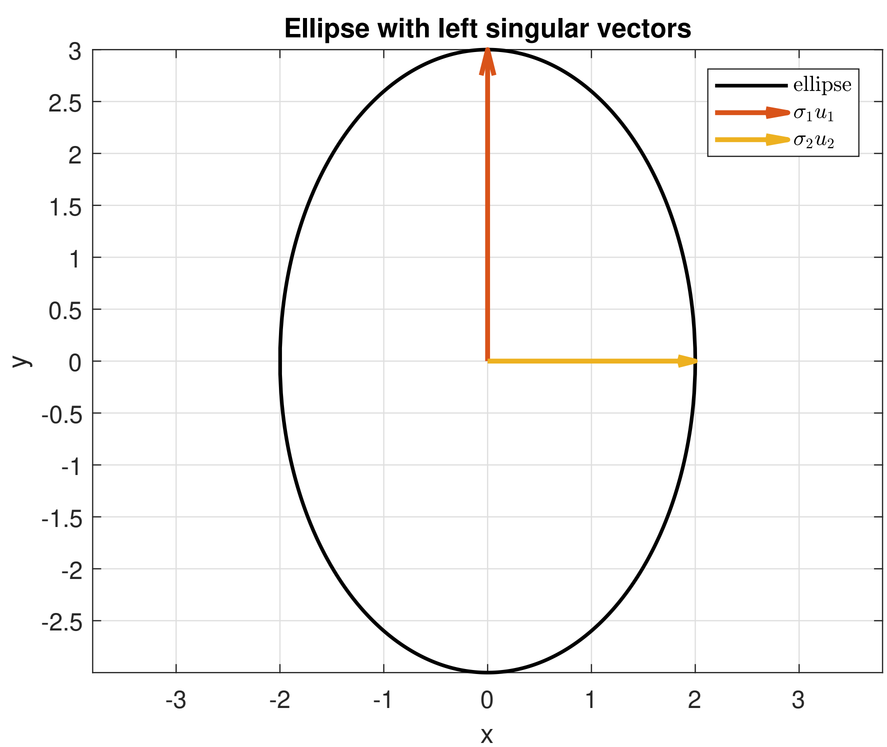
(a)



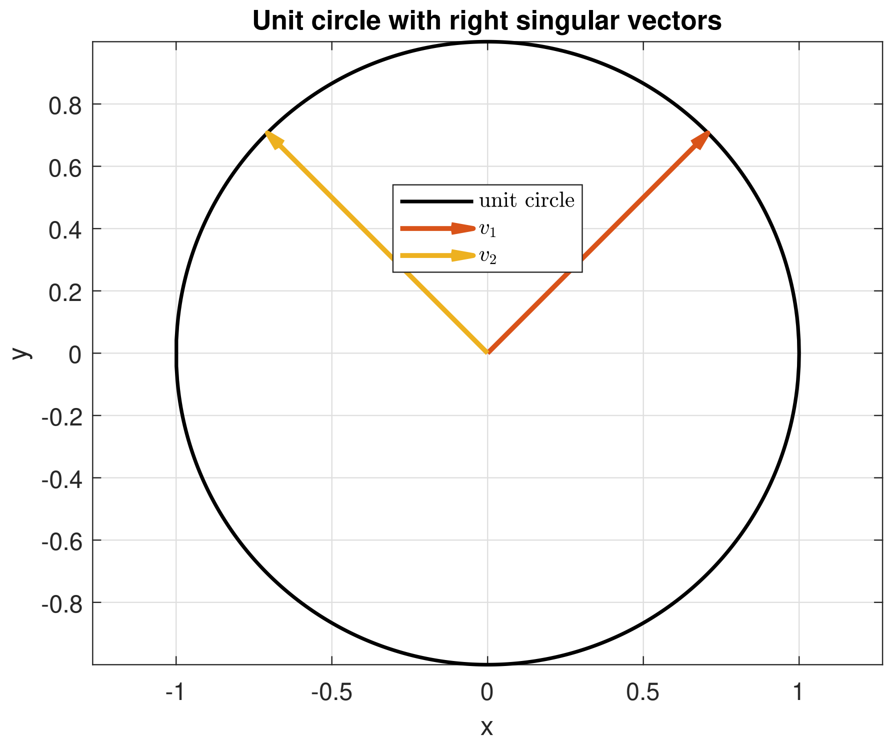


(b)

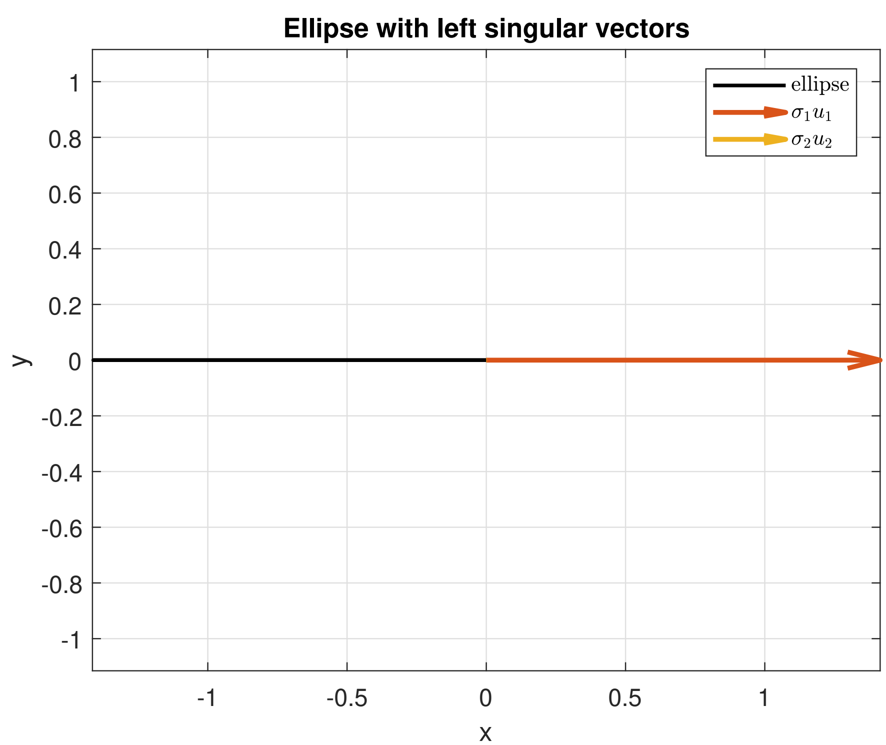

Unit circle with right singular vectors
unit circle
v_1
v_2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
y
-1
-0.5
0
0.5
1
x


Ellipse with left singular vectors
ellipse
$\sigma_1 u_1$
$\sigma_2 u_2$
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
y
-3
-2
-1
0
1
2
3
x

(d)

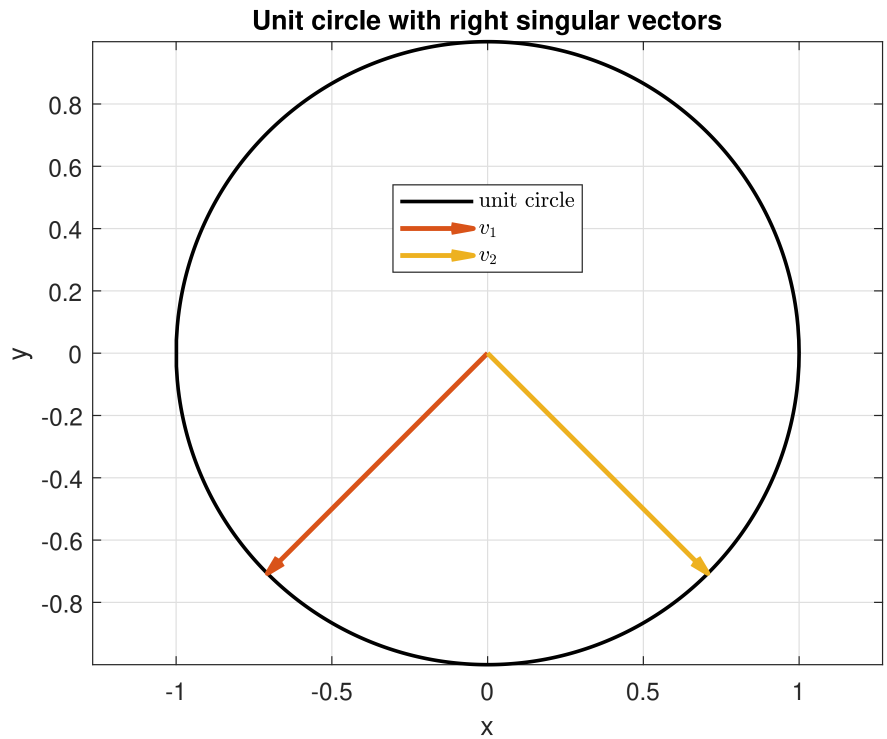


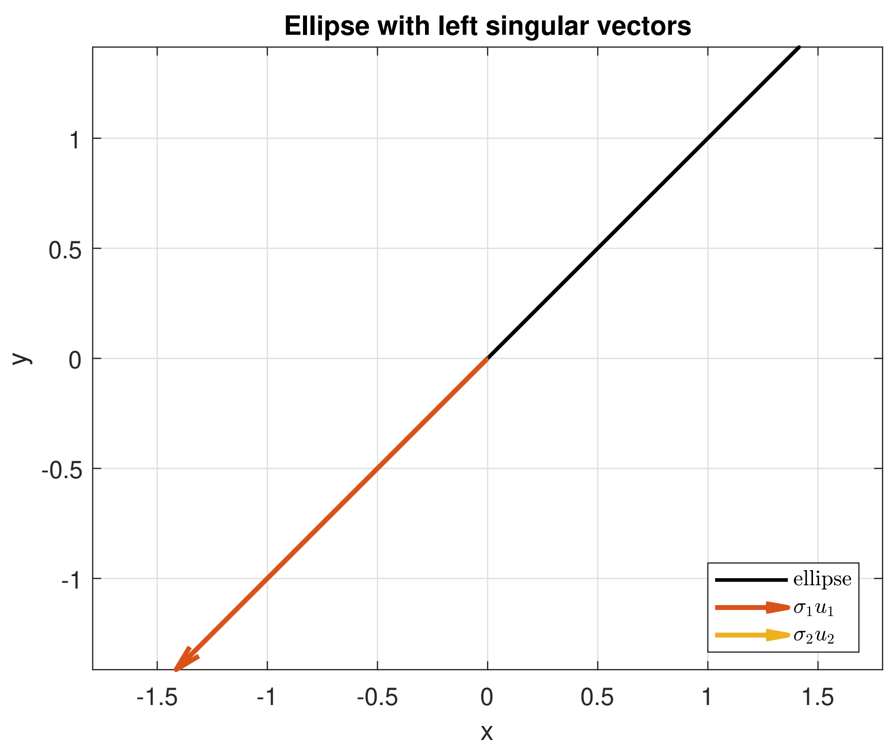
Unit circle with right singular vectors
unit circle
v_1
v_2
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
y
-1
-0.5
0
0.5
1
x



Ellipse with left singular vectors
ellipse
$\sigma_1 u_1$
$\sigma_2 u_2$
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
y
-1
-0.5
0
0.5
1
x

(e)


Unit circle with right singular vectors
unit circle
v_1
v_2
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
y
-1
-0.5
0
0.5
1
x


Ellipse with left singular vectors
ellipse
$\sigma_1 u_1$
$\sigma_2 u_2$
1
0.5
0
-0.5
-1
y
-1.5
-1
-0.5
0
0.5
1
1.5
x